

# Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration

Jing Wu<sup>1</sup>, Yan-Rui Liu<sup>1\*</sup>

<sup>1</sup>*School of Physics and Key Laboratory of Particle Physics and Particle Irradiation (MOE), Shandong University, Jinan 250100, China*

Kan Chen<sup>2,3</sup>, Xiang Liu<sup>2,3†</sup>

<sup>2</sup>*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*

<sup>3</sup>*Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China*

Shi-Lin Zhu<sup>4,5,6‡</sup>

<sup>4</sup>*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

<sup>5</sup>*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

<sup>6</sup>*Center of High Energy Physics, Peking University, Beijing 100871, China*

In the framework of the color-magnetic interaction, we systematically investigate the mass spectrum of the tetraquark states composed of four heavy quarks with the  $QQ\bar{Q}\bar{Q}$  configuration in this work. We also discuss their strong decay patterns, which shall be helpful to the experimental search for these exotic states.

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## I. INTRODUCTION

Searching for exotic states is an interesting research field full of challenges and opportunities. Among the various exotic candidates, the multi-quark state is a very popular hadronic configuration. In fact, the concept of the multi-quark state was proposed long time ago in Refs. [1–3]. In the past decades, experimentalists have been trying to find them. The observations of the charmonium-like/bottomonium-like  $XYZ$  states [4–10] have provided valuable hints of the existence of the exotic states. Especially, these charged charmonium-like or bottomonium-like states like the  $Z(4430)$  [11–14], the  $Z_1(4050)$  [15], the  $Z_2(4250)$  [15], the  $Z_c(3900)$  [16–19], the  $Z_c(3885)$  [20–22], the  $Z_c(4020)$  [23, 24], the  $Z_c(4025)$  [25, 26], the  $Z_c(4200)$  [13], the  $Z_b(10610)$  [27] and the  $Z_b(10650)$  [27] have forced us to consider the existence of the multi-quark matter very seriously. So many observations in the heavy quark sector in recent years are surprising, which provides new opportunities for us to understand the nature of strong interaction.

Experimentalists continue to surprise us after the observations of  $XYZ$  states. In 2015, the LHCb Collaboration reported the hidden-charm pentaquarks  $P_c(4380)$  and  $P_c(4450)$  in the invariant mass of  $\psi p$  [1]. They are consistent with the pentaquark structures [28–36]. With a model-independent analysis, the LHCb confirms their previous model-dependent evidence for these states [37]. Very recently, a structure  $X(5568)$  was announced by the DØ Collaboration [38]. This narrow state is about 200 MeV below the  $B\bar{K}$  threshold and decays into  $B_s^0\pi^\pm$ . Therefore, the  $X(5568)$  may be a typical tetraquark state composed of four different flavors [39–51] while the molecule state assignment to  $X(5568)$  is not favored [52]. However, the LHCb Collaboration did not confirm the existence of the  $X(5568)$  [53], which makes some theorists consider the difficulty of explaining the  $X(5568)$  as a genuine resonance [54–62]. Although there exist different opinions of the  $X(5568)$ , the observation of the  $X(5568)$  again ignites theorist's enthusiasm of exploring exotic tetraquark states.

As discussed above, there exist possible candidates of the hidden-charm tetraquark states and the tetraquark with a heavy flavor quark and three light quarks. If the multi-quark states indeed exist in nature, we have a strong reason to believe that there are more tetraquark states with other flavor configurations. In this work, we focus on the heavy-flavor tetraquark states with the  $Q_1Q_2\bar{Q}_3\bar{Q}_4$  configuration, where  $Q_i$  is a  $c$  or  $b$  quark. Although these tetraquark states are still missing in experiment, it is time to carry out a systematic investigation of the mass spectrum of the  $Q_1Q_2\bar{Q}_3\bar{Q}_4$  tetraquark system, which may provide important information to further experimental exploration.

\*Electronic address: [yrlu@sdu.edu.cn](mailto:yrlu@sdu.edu.cn)

†Electronic address: [xiangliu@lzu.edu.cn](mailto:xiangliu@lzu.edu.cn)

‡Electronic address: [zhusl@pku.edu.cn](mailto:zhusl@pku.edu.cn)

[1] For convenience, here and in the following,  $\psi$  means  $J/\psi$  once the symbol is adopted.

In Ref. [63], Iwasaki studied the hidden-charm tetraquark state composed of a pair of charmed quark and anti-quark as well as a pair of light quark and anti-quark, which has  $c\bar{c}q\bar{q}$  configuration. The dimeson configuration ( $Q^2\bar{q}^2$ ) is stable against strong decay into two mesons [64]. Chao performed a systematical investigation of the  $cc\bar{c}\bar{c}$  tetraquark system in a quark-gluon model for the first time in Ref. [65]. In Ref. [66], the author used the Born-Oppenheimer approximation for heavy quarks in the MIT bag, and found that the heavy-quark system  $c^2\bar{c}^2$  is stable against breakup into two  $c\bar{c}$  pairs. But in a potential model calculation [67], the authors suggested that for identical quarks, there is no stable  $QQ\bar{Q}\bar{Q}$  state. Similar opinions were shared by the authors of Ref. [68]. However, Lloyd and Vary adopted a parameterized Hamiltonian to compute the spectrum of the  $cc\bar{c}\bar{c}$  tetraquark states [69]. The tetraquark spectrum was also studied with a generalization of the hyperspherical harmonic formalism in Ref. [70]. In addition, the calculation of the chromomagnetic interaction for the  $Q^2\bar{Q}^2$  system was performed in Ref. [71]. In understanding the nature of the  $Y(4260)$  meson in Ref. [72], as a byproduct, a lattice study indicates that the  $J^{PC} = 1^{--} cc\bar{c}\bar{c}$  state is possible. If the P-wave  $cc\bar{c}\bar{c}$  state exists where the orbital angular momentum contributes some repulsion, the lower ground tetraquark states should also exist.

In this work, we calculate the mass splitting of the  $Q_1Q_2\bar{Q}_3\bar{Q}_4$  tetraquark system in a simple quark model systematically. A typical feature for such tetraquarks is that the isospin is always zero and the flavor wave function is always symmetric for identical quarks. One may only focus on the color-spin part when the Pauli principle is employed to exclude some configurations. For the interaction between the heavy quarks, the short-range gluon exchange force is a dominant source. In the one-gluon-exchange potential model, the color-spin or color-magnetic interaction part is responsible for the mass splitting of the ground hadrons with the same flavor content. In the present study, we will adopt the color-magnetic interaction (CMI) to perform the calculations, which violates the heavy quark symmetry.

We organize the paper as follows. After the introduction, we present the formalism of calculation in Sec. II. We give the numerical results and discuss the possible decay modes in Sec. III. We summarize our results in the final section.

## II. FORMALISM

The color-magnetic interaction reads

$$H_{CM} = - \sum_{i,j} C_{ij} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j, \quad (1)$$

where  $\lambda_i$ 's are the Gell-Mann matrices and  $\sigma_i$ 's are the Pauli matrices. The above Hamiltonian was deduced from the one-gluon-exchange interaction [73]. The effective coupling constants  $C_{ij}$  incorporate effects from the spatial wave function and the quark masses, which depend on the system. We determine their values in the next section. This Hamiltonian leads to a mass formula for the studied system

$$M = \sum_i m_i + \langle H_{CM} \rangle, \quad (2)$$

where  $m_i$  is the effective mass of the  $i$ -th constituent quark, which includes the constituent quark mass and effects from other terms such as color-electric interaction and color confinement.

To calculate the matrix elements for the color-spin interaction, one may construct the *color*  $\otimes$  *spin* wave functions explicitly and calculate them by definition. In studying multiquark systems, a simpler way was used in Refs. [74–76]. One just calculates the matrix elements in color space and in spin space separately with the Hamiltonians  $H_C = - \sum_{i,j} C_{ij} \lambda_i \cdot \lambda_j$  and  $H_S = - \sum_{i,j} C_{ij} \sigma_i \cdot \sigma_j$ . Then  $\langle H_{CM} \rangle$  is obtained after a kind of “tensor product” of  $\langle H_C \rangle$  and  $\langle H_S \rangle$  is performed.

In order to consider the constraint from the Pauli principle, we use a diquark-antidiquark picture to analyze the configurations. In the spin space, the allowed wave functions read

$$\begin{aligned} \chi_1 &= |(Q_1Q_2)_1(\bar{Q}_3\bar{Q}_4)_1\rangle_2, & \chi_2 &= |(Q_1Q_2)_1(\bar{Q}_3\bar{Q}_4)_1\rangle_1, \\ \chi_3 &= |(Q_1Q_2)_1(\bar{Q}_3\bar{Q}_4)_1\rangle_0, & \chi_4 &= |(Q_1Q_2)_1(\bar{Q}_3\bar{Q}_4)_0\rangle_1, \\ \chi_5 &= |(Q_1Q_2)_0(\bar{Q}_3\bar{Q}_4)_1\rangle_1, & \chi_6 &= |(Q_1Q_2)_0(\bar{Q}_3\bar{Q}_4)_0\rangle_0, \end{aligned} \quad (3)$$

where the subscripts on the right hand side denote the spin of the  $Q_1Q_2$ ,  $\bar{Q}_3\bar{Q}_4$ , and that of the system respectively. According to the  $SU(3)$  group theory, the diquark in the color space belongs to the representation  $6_c$  or  $\bar{3}_c$ , while the anti-diquark's representation is  $\bar{6}_c$  or  $3_c$ . Then one has two kinds of color-singlet state

$$\phi_1 = |(Q_1Q_2)^6(\bar{Q}_3\bar{Q}_4)^{\bar{6}}\rangle, \quad \phi_2 = |(Q_1Q_2)^{\bar{3}}(\bar{Q}_3\bar{Q}_4)^3\rangle. \quad (4)$$

Combine the spin and color wave functions, we get twelve possible  $color \otimes spin$  wave functions for the  $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$  system

$$\begin{aligned}
\phi_1 \chi_1 &= |(Q_1 Q_2)_1^6 (\bar{Q}_3 \bar{Q}_4)_1^{\bar{6}}\rangle_2 \delta_{12} \delta_{34}, & \phi_2 \chi_1 &= |(Q_1 Q_2)_1^{\bar{3}} (\bar{Q}_3 \bar{Q}_4)_1^3\rangle_2, \\
\phi_1 \chi_2 &= |(Q_1 Q_2)_1^6 (\bar{Q}_3 \bar{Q}_4)_1^{\bar{6}}\rangle_1 \delta_{12} \delta_{34}, & \phi_2 \chi_2 &= |(Q_1 Q_2)_1^{\bar{3}} (\bar{Q}_3 \bar{Q}_4)_1^3\rangle_1, \\
\phi_1 \chi_3 &= |(Q_1 Q_2)_1^6 (\bar{Q}_3 \bar{Q}_4)_1^{\bar{6}}\rangle_0 \delta_{12} \delta_{34}, & \phi_2 \chi_3 &= |(Q_1 Q_2)_1^{\bar{3}} (\bar{Q}_3 \bar{Q}_4)_1^3\rangle_0, \\
\phi_1 \chi_4 &= |(Q_1 Q_2)_1^6 (\bar{Q}_3 \bar{Q}_4)_0^{\bar{6}}\rangle_1 \delta_{12}, & \phi_2 \chi_4 &= |(Q_1 Q_2)_1^{\bar{3}} (\bar{Q}_3 \bar{Q}_4)_0^3\rangle_1 \delta_{34}, \\
\phi_1 \chi_5 &= |(Q_1 Q_2)_0^6 (\bar{Q}_3 \bar{Q}_4)_1^{\bar{6}}\rangle_1 \delta_{34}, & \phi_2 \chi_5 &= |(Q_1 Q_2)_0^{\bar{3}} (\bar{Q}_3 \bar{Q}_4)_1^3\rangle_1 \delta_{12}, \\
\phi_1 \chi_6 &= |(Q_1 Q_2)_0^6 (\bar{Q}_3 \bar{Q}_4)_0^{\bar{6}}\rangle_0, & \phi_2 \chi_6 &= |(Q_1 Q_2)_0^{\bar{3}} (\bar{Q}_3 \bar{Q}_4)_0^3\rangle_0 \delta_{12} \delta_{34},
\end{aligned} \tag{5}$$

where the used notation is  $|(Q_1 Q_2)_{spin}^{color} (\bar{Q}_3 \bar{Q}_4)_{spin}^{color}\rangle_{spin}$ . We insert a symbol  $\delta_{ij}$  in the wave functions to reflect the constraint from the Pauli principle. When the  $i$ -th quark and the  $j$ -th quark are the same,  $\delta_{ij} = 0$ . Otherwise,  $\delta_{ij} = 1$ .

Replacing  $Q_i$  with  $c$  or  $b$  quark, one gets nine possibilities for the flavor content, six of which need to be studied:  $bb\bar{b}\bar{b}$ ,  $cc\bar{c}\bar{c}$ ,  $bb\bar{c}\bar{c}$ ,  $bb\bar{c}\bar{b}$ ,  $cc\bar{c}\bar{b}$  and  $bc\bar{b}\bar{c}$ . The other three cases  $cc\bar{b}\bar{b}$ ,  $cbb\bar{b}$ , and  $bcc\bar{c}$  correspond to antiparticles of  $bb\bar{c}\bar{c}$ ,  $bb\bar{b}\bar{c}$ , and  $cc\bar{c}\bar{b}$ , respectively. So their formulas are not independent. Here, considering the Pauli principle, one may categorize the six systems into three sets: (1)  $bb\bar{b}\bar{b}$ ,  $cc\bar{c}\bar{c}$  and  $bb\bar{c}\bar{c}$ , where  $\delta_{12} = \delta_{34} = 0$ ; (2)  $bb\bar{b}\bar{c}$  and  $cc\bar{b}\bar{c}$ , where  $\delta_{12} = 0, \delta_{34} = 1$ ; and (3)  $bc\bar{b}\bar{c}$  where  $\delta_{12} = \delta_{34} = 1$ . The number of independent wave functions for them is 4, 6, and 12, respectively. We present the resulting CMI matrix elements system by system. By the way, the usually mentioned ‘‘good’’ diquark exists only in the systems containing the  $(bc)$  substructure since the most attractive  $(bb)_{spin-0}^{\bar{3}_c}$  and  $(cc)_{spin-0}^{\bar{3}_c}$  objects are forbidden by the Pauli principle.

### A. The $bb\bar{b}\bar{b}$ and $cc\bar{c}\bar{c}$ systems

The color-spin structures for these two systems are the same. The only difference lies in the quark mass. So we put them together for discussions.

The  $bb\bar{b}\bar{b}$  system is a neutral state. Its possible quantum numbers are  $I^G(J^{PC}) = 0^+(2^{++})$ ,  $0^-(1^{+-})$ , or  $0^+(0^{++})$ . The number of states is constrained by the Pauli principle. For the case  $J = 2$ , the wave function is  $\phi_2 \chi_1$  and the obtained  $\langle H_{CM} \rangle$  is given by  $\frac{16}{3}(C_{bb} + C_{\bar{b}\bar{b}})$ . The color-magnetic interaction is certainly repulsive. For the case  $J = 1$ , the wave function is  $\phi_2 \chi_2$  and  $\langle H_{CM} \rangle = \frac{16}{3}(C_{bb} - C_{\bar{b}\bar{b}})$ . Since the quark-quark interaction and the quark-antiquark interaction is related with the G-parity transformation, the CMI for this  $J = 1$  state is expected to be weak. For the case  $J = 0$ , there are two possible wave functions  $\phi_2 \chi_3$  and  $\phi_1 \chi_6$ . Although the color-spin structures are different, their mixing is allowed by the color-magnetic interaction. The obtained symmetric CMI matrix in the  $(\phi_2 \chi_3, \phi_1 \chi_6)^T$  base is

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{16}{3}(C_{bb} - 2C_{\bar{b}\bar{b}}) & 8\sqrt{6}C_{\bar{b}\bar{b}} \\ & 8C_{\bar{b}\bar{b}} \end{pmatrix}. \tag{6}$$

The  $cc\bar{c}\bar{c}$  system has similar expressions. For the case  $J = 2$ ,  $\langle H_{CM} \rangle = \frac{16}{3}(C_{cc} + C_{\bar{c}\bar{c}})$  with the color-spin wave function  $\phi_2 \chi_1$ . For the case  $J = 1$ ,  $\langle H_{CM} \rangle = \frac{16}{3}(C_{cc} - C_{\bar{c}\bar{c}})$  with the wave function  $\phi_2 \chi_2$ . For  $J = 0$ , the CMI matrix is

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{16}{3}(C_{cc} - 2C_{\bar{c}\bar{c}}) & 8\sqrt{6}C_{\bar{c}\bar{c}} \\ & 8C_{\bar{c}\bar{c}} \end{pmatrix}. \tag{7}$$

The above compact tetraquark system has the same quark content as the molecular states composed of two bottomonium (charmonium) mesons. However, the interaction between heavy quarks is dominantly through the short-range gluon exchange. Once the interaction is strong enough to bind the two mesons, the resulting object very probably tends to form a compact structure instead of a loosely-bound molecule.

In the extreme case that the molecules exist, the configuration mixing is possible. For the  $S$ -wave  $\Upsilon\Upsilon$  ( $\psi\psi$ ) state, the allowed quantum numbers are  $I^G(J^{PC}) = 0^+(2^{++})$  or  $0^+(0^{++})$  since the wave function in spin space should be symmetric for identical mesons. The quantum numbers for the  $S$ -wave state composed of two  $\eta_b$  ( $\eta_c$ ) mesons are only  $0^+(0^{++})$ . For the state composed of one  $\eta_b$  ( $\eta_c$ ) and one  $\Upsilon$  ( $\psi$ ), the quantum numbers are just  $I^G(J^{PC}) = 0^-(1^{+-})$ . Therefore, the allowed  $J^{PC}$  appear both in the molecule and diquark-antidiquark picture. The number of states is also equivalent. Considering that the interaction between heavy quarks is through the gluon exchange force, one does not expect large mass difference between these two configurations. The compact structure may contribute significantly to the properties of the molecules with the same quantum numbers.

If the tetraquark states have larger masses, the quark rearrangements into the meson-meson channels with the same quantum numbers may happen. We will discuss such possible decay patterns after their masses are estimated.

### B. The $b\bar{b}c\bar{c}$ and $c\bar{c}b\bar{b}$ systems

The two systems are related through the  $C$ -parity transformation and they have the same color-spin matrix elements. The possible quantum numbers of them are  $I(J^P) = 0(2^+)$ ,  $0(1^+)$ , or  $0(0^+)$ . Again the Pauli principle results in four states for the  $b\bar{b}c\bar{c}$  (or  $c\bar{c}b\bar{b}$ ) system. The color-spin wave functions are  $\phi_2\chi_1$  and  $\phi_2\chi_2$  for the case of  $J = 2$  and  $J = 1$ , respectively. The corresponding  $\langle H_{CM} \rangle$ 's are  $\frac{8}{3}(C_{bb} + C_{cc} + 2C_{b\bar{c}})$  and  $\frac{8}{3}(C_{bb} + C_{cc} - 2C_{b\bar{c}})$ . For the case  $J = 0$ , the allowed wave functions are the same as those of the previous systems,  $\phi_2\chi_3$  and  $\phi_1\chi_6$ . Consider their mixing, one gets

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{8}{3}(C_{bb} + C_{cc} - 4C_{b\bar{c}}) & 8\sqrt{6}C_{b\bar{c}} \\ 4(C_{bb} + C_{cc}) \end{pmatrix}. \quad (8)$$

These systems have the same quark content as the  $B_c^{(*)}B_c^{(*)}$  meson-meson states. The quantum numbers of  $B_c^{*-}B_c^{*-}$  ( $B_c^{*+}B_c^{*+}$ ) are  $I(J^P) = 0(2^+)$  or  $0(0^+)$ , those of  $B_c^-B_c^-$  ( $B_c^+B_c^+$ ) are  $0(0^+)$ . There is only one  $B_c^-B_c^{*-}$  ( $B_c^+B_c^{*+}$ ) state with  $I(J^P) = 0(1^+)$ .

### C. The $b\bar{b}b\bar{c}$ and $c\bar{c}b\bar{b}$ systems

The two systems have the same matrix elements. Their quantum numbers are also  $I(J^P) = 0(2^+)$ ,  $0(1^+)$ , or  $0(0^+)$ . We here focus on the  $b\bar{b}b\bar{c}$  system. Now the number of the vector states is 3 and that of the scalar states is 2. For the  $J = 2$  case, the color-spin wave function is again  $\phi_2\chi_1$ . The resulting color-magnetic matrix element is  $\langle H_{CM} \rangle = \frac{8}{3}(C_{bb} + C_{bc} + C_{b\bar{b}} + C_{b\bar{c}})$ . For the case  $J = 1$ , three possible wave functions  $\phi_2\chi_2$ ,  $\phi_2\chi_4$ , and  $\phi_1\chi_5$  are allowed. The last one has different color structure from the other two. Although their mixing occurs, from the obtained matrix [base:  $(\phi_2\chi_2, \phi_2\chi_4, \phi_1\chi_5)^T$ ]

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{8}{3}(C_{bb} + C_{bc} - C_{b\bar{b}} - C_{b\bar{c}}) & \frac{8\sqrt{2}}{3}(C_{b\bar{b}} - C_{b\bar{c}}) & 8(C_{b\bar{c}} - C_{b\bar{b}}) \\ \frac{8}{3}(C_{bb} - 3C_{bc}) & -4\sqrt{2}(C_{b\bar{c}} + C_{b\bar{b}}) & \frac{4}{3}(3C_{bb} - C_{bc}) \end{pmatrix}, \quad (9)$$

one observes that the mixing strength for  $\phi_2\chi_2$  and  $\phi_2\chi_4$  and that for  $\phi_2\chi_2$  and  $\phi_1\chi_5$  may be both small. The remaining case is for  $J = 0$ , where possible wave functions are  $\phi_2\chi_3$  and  $\phi_1\chi_6$ . Now, one has

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{8}{3}(C_{bb} + C_{bc} - 2C_{b\bar{b}} - 2C_{b\bar{c}}) & 4\sqrt{6}(C_{b\bar{b}} + C_{b\bar{c}}) \\ 4(C_{bb} + C_{bc}) \end{pmatrix}. \quad (10)$$

The meson-meson systems with the quark content  $b\bar{b}b\bar{c}$  are  $\Upsilon B_c^-$ ,  $\Upsilon B_c^{*-}$ ,  $\eta_b B_c^-$ , and  $\eta_b B_c^{*-}$ . Their quantum numbers are  $I(J^P) = 0(1^+)$ ,  $0([2, 1, 0]^+)$ ,  $0(0^+)$ , and  $0(1^+)$ , respectively.

### D. The $c\bar{c}c\bar{b}$ and $b\bar{c}c\bar{c}$ systems

The situation is similar to the  $b\bar{b}b\bar{c}$  and  $c\bar{c}b\bar{b}$  systems. By exchanging  $b$  and  $c$  there, one easily gets relevant matrix elements. For comparison, we focus on the  $c\bar{c}c\bar{b}$  system. For the case  $J = 2$ , one has  $\langle H_{CM} \rangle = \frac{8}{3}(C_{cc} + C_{bc} + C_{b\bar{c}} + C_{c\bar{c}})$ . For the case  $J = 1$ , the matrix for the color-spin interaction reads

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{8}{3}(C_{cc} + C_{bc} - C_{b\bar{c}} - C_{c\bar{c}}) & \frac{8\sqrt{2}}{3}(C_{b\bar{c}} - C_{c\bar{c}}) & 8(C_{c\bar{c}} - C_{b\bar{c}}) \\ \frac{8}{3}(C_{cc} - 3C_{bc}) & -4\sqrt{2}(C_{c\bar{c}} + C_{b\bar{c}}) & \frac{4}{3}(3C_{cc} - C_{bc}) \end{pmatrix}. \quad (11)$$

For the case  $J = 0$ , the matrix is

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{8}{3}(C_{cc} + C_{bc} - 2C_{b\bar{c}} - 2C_{c\bar{c}}) & 4\sqrt{6}(C_{b\bar{c}} + C_{c\bar{c}}) \\ 4(C_{cc} + C_{bc}) \end{pmatrix}. \quad (12)$$

The signs for the non-diagonal matrix elements seem to be inconsistent with the previous systems after the replacements  $b \rightarrow c$ , and  $c \rightarrow b$ . Actually they do not affect the final results. For detailed argument, one may consult Eq. (2) of Ref. [77] and relevant explanations there.

The meson-meson states that these tetraquarks can rearrange into are  $\psi B_c$ ,  $\psi B_c^*$ ,  $\eta_c B_c$ , and  $\eta_c B_c^*$ .

### E. The $b\bar{c}b\bar{c}$ system

The Pauli principle does not give any constraints for the wave functions now. The two wave functions for  $J = 2$  and the four wave functions for  $J = 0$  will mix, respectively. However, one should be careful in discussing the mixing with the six wave functions for  $J = 1$  because the system is neutral and may have  $C$ -parity.

If  $J = 2$ , both the diquark and the antidiquark have angular momentum 1. The state should have definite  $C$ -parity + and the quantum numbers are  $I^G(J^{PC}) = 0^+(2^{++})$ . With the base  $(\phi_1\chi_1, \phi_2\chi_1)^T$ , one may get the CMI matrix

$$\langle H_{CM} \rangle = \begin{pmatrix} -\frac{2}{3}(4C_{bc} - 5C_{b\bar{b}} - 10C_{b\bar{c}} - 5C_{c\bar{c}}) & 2\sqrt{2}(C_{b\bar{b}} - 2C_{b\bar{c}} + C_{c\bar{c}}) \\ \frac{4}{3}(4C_{bc} + C_{b\bar{b}} + 2C_{b\bar{c}} + C_{c\bar{c}}) & \end{pmatrix}. \quad (13)$$

If  $J = 0$ , both the diquark and the antidiquark have the same angular momentum. The quantum numbers for the system are  $I^G(J^{PC}) = 0^+(0^{++})$ . The obtained CMI matrix is

$$\langle H_{CM} \rangle = \begin{pmatrix} -\frac{4}{3} \begin{pmatrix} 2C_{bc} + 5C_{b\bar{b}} \\ +10C_{b\bar{c}} + 5C_{c\bar{c}} \end{pmatrix} & -\frac{10}{\sqrt{3}}(C_{b\bar{b}} - 2C_{b\bar{c}} + C_{c\bar{c}}) & 4\sqrt{2}(C_{b\bar{b}} - 2C_{b\bar{c}} + C_{c\bar{c}}) & 2\sqrt{6}(C_{b\bar{b}} + 2C_{b\bar{c}} + C_{c\bar{c}}) \\ 8C_{bc} & 2\sqrt{6}(C_{b\bar{b}} + 2C_{b\bar{c}} + C_{c\bar{c}}) & 0 & 0 \\ \frac{8}{3}(2C_{bc} - C_{b\bar{b}} - 2C_{b\bar{c}} - C_{c\bar{c}}) & -\frac{4}{\sqrt{3}}(C_{b\bar{b}} - 2C_{b\bar{c}} + C_{c\bar{c}}) & -16C_{bc} & \end{pmatrix}, \quad (14)$$

where the base is  $(\phi_1\chi_3, \phi_1\chi_6, \phi_2\chi_3, \phi_2\chi_6)^T$ .

If  $J = 1$ , the states  $\phi_1\chi_2$  and  $\phi_2\chi_2$  have negative  $C$ -parities. All the other four wave functions  $\phi_1\chi_4$ ,  $\phi_2\chi_4$ ,  $\phi_1\chi_5$ , and  $\phi_2\chi_5$  are not invariant under  $C$ -parity transformation. But we may construct four states which are invariant under  $C$ -parity transformations. The basic procedure is similar to that given in Ref. [78]. Explicitly, the two  $C = +$  states are

$$\begin{aligned} [\phi\chi]_+^{6\bar{6}} &= \frac{1}{\sqrt{2}}(\phi_1\chi_4 + \phi_1\chi_5), \\ [\phi\chi]_+^{\bar{3}3} &= \frac{1}{\sqrt{2}}(\phi_2\chi_4 + \phi_2\chi_5), \end{aligned} \quad (15)$$

and the two  $C = -$  states are

$$\begin{aligned} [\phi\chi]_-^{6\bar{6}} &= \frac{1}{\sqrt{2}}(\phi_1\chi_4 - \phi_1\chi_5), \\ [\phi\chi]_-^{\bar{3}3} &= \frac{1}{\sqrt{2}}(\phi_2\chi_4 - \phi_2\chi_5). \end{aligned} \quad (16)$$

Only states with the same quantum numbers may mix. So we have two color-spin matrices. For the states with  $I^G(J^{PC}) = 0^+(1^{+-})$ , the matrix is

$$\langle H_{CM} \rangle = \begin{pmatrix} \frac{2}{3}(4C_{bc} + 5C_{b\bar{b}} + 5C_{c\bar{c}} - 10C_{b\bar{c}}) & -2\sqrt{2}(C_{b\bar{b}} + C_{c\bar{c}} + 2C_{b\bar{c}}) \\ \frac{4}{3}(-4C_{bc} + C_{b\bar{b}} + C_{c\bar{c}} - 2C_{b\bar{c}}) & \end{pmatrix}, \quad (17)$$

with the base  $([\phi\chi]_+^{6\bar{6}}, [\phi\chi]_+^{\bar{3}3})^T$ . For the states with  $I^G(J^{PC}) = 0^-(1^{+-})$ , the matrix reads

$$\langle H_{CM} \rangle = \begin{pmatrix} -\frac{2}{3} \begin{pmatrix} 4C_{bc} + 5C_{b\bar{b}} \\ +5C_{c\bar{c}} + 10C_{b\bar{c}} \end{pmatrix} & 2\sqrt{2}(C_{b\bar{b}} + C_{c\bar{c}} - 2C_{b\bar{c}}) & \frac{20}{3}(C_{b\bar{b}} - C_{c\bar{c}}) & -4\sqrt{2}(C_{b\bar{b}} - C_{c\bar{c}}) \\ \frac{4}{3} \begin{pmatrix} 4C_{bc} - C_{b\bar{b}} \\ -C_{c\bar{c}} - 2C_{b\bar{c}} \end{pmatrix} & -4\sqrt{2}(C_{b\bar{b}} - C_{c\bar{c}}) & \frac{8}{3}(C_{b\bar{b}} - C_{c\bar{c}}) & \\ \frac{2}{3} \begin{pmatrix} 4C_{bc} - 5C_{b\bar{b}} \\ -5C_{c\bar{c}} + 10C_{b\bar{c}} \end{pmatrix} & 2\sqrt{2}(C_{b\bar{b}} + C_{c\bar{c}} + 2C_{b\bar{c}}) & & \\ & & -\frac{4}{3} \begin{pmatrix} 4C_{bc} + C_{b\bar{b}} \\ +C_{c\bar{c}} - 2C_{b\bar{c}} \end{pmatrix} & \end{pmatrix}, \quad (18)$$

where the base is  $(\phi_1\chi_2, \phi_2\chi_2, [\phi\chi]_{-}^{6\bar{6}}, [\phi\chi]_{-}^{3\bar{3}})^T$ .

There are two kinds of molecule configurations for the  $b\bar{c}\bar{b}\bar{c}$  system. In the bottomonium+charmonium case, the allowed quantum numbers are  $I^G(J^{PC}) = 0^+(0^{++})$  for the  $\eta_b\eta_c$  system,  $0^-(1^{+-})$  for the  $\eta_b\psi$  system,  $0^-(1^{+-})$  for the  $\Upsilon\eta_c$ , and  $0^+([2, 1, 0]^{++})$  for the  $\Upsilon\psi$  system. In the meson-antimeson case, those for  $B_c^-B_c^+$  are  $0^+(0^{++})$ , those for  $B_c^-B_c^{*+} \pm B_c^{*-}B_c^+$  are  $0^\pm(1^{\pm\pm})$ , and those for  $B_c^{*-}B_c^{*+}$  are  $0^+([2, 0]^{++})$  or  $0^-(1^{+-})$ .

### III. NUMERICAL RESULTS AND DISCUSSIONS

#### A. Parameters

We need to determine six coefficients  $C_{b\bar{b}}$ ,  $C_{c\bar{c}}$ ,  $C_{b\bar{c}}$ ,  $C_{bb}$ ,  $C_{cc}$ , and  $C_{bc}$  in discussing the mass splittings for various  $Q_1Q_2\bar{Q}_3\bar{Q}_4$  systems. Their masses may be further estimated with the Hamiltonian in Eq. (2) once the effective quark masses  $m_c$  and  $m_b$  are used.

From the mass splitting between  $\Upsilon$  and  $\eta_b$ ,  $m_\Upsilon - m_{\eta_b} = [\frac{16}{3}C_{b\bar{b}}] - [-16C_{b\bar{b}}] = 64$  MeV [79], one extracts  $C_{b\bar{b}} = 3.0$  MeV. Similarly, the value of  $C_{c\bar{c}} = 5.3$  MeV is obtained from the mass splitting  $m_{J/\psi} - m_{\eta_c} = 114$  MeV. Since the excited  $B_c^*$  meson has not been observed yet, we just estimate the value of  $C_{b\bar{c}}$  to be 3.3 MeV from  $m_{B_c^*} - m_{B_c} = 70$  MeV calculated with a quark model [80]. For the extraction of the other three parameters, there are still no available baryon masses from experiments. Even the lowest  $\Xi_{cc}$  is not confirmed. Here we just perform our calculation with the approximation  $C_{bb} = C_{b\bar{b}}$ ,  $C_{cc} = C_{c\bar{c}}$ , and  $C_{bc} = C_{b\bar{c}}$ .

To determine the masses of the tetraquarks, we adopt two approaches in the present work: 1) One estimates the meson masses with effective heavy quarks,  $m_c = 1430$  MeV and  $m_b = 4630$  MeV. These values were adopted in understanding the strange properties of  $D_{sJ}(2632)$  [81] and  $X(5568)$  [45]; 2) One calculates the masses from a meson-meson threshold, where the relevant meson masses are:  $m_\Upsilon = 9460.3$  MeV,  $m_\psi = 3096.9$  MeV, and  $m_{B_c} = 6275.6$  MeV [79]. The latter method has been used in estimating the mass of an exotic  $T_{cc}$  [82].

#### B. The $b\bar{b}\bar{b}\bar{b}$ and $c\bar{c}\bar{c}\bar{c}$ systems

It is easy to get the numerical results for the CMI matrix elements with the estimated parameters. We present in Table I the CMI matrices, their eigenvalues and corresponding eigenvectors, and estimated masses with two different approaches. The mass splitting between these tetraquark states with different spins is at most 125 MeV for the bottom system and 220 MeV for the charmed system. However, the mass difference in the two approaches is around 370 MeV for the bottom system and around 420 MeV for the charmed system.

TABLE I: Results for the  $b\bar{b}\bar{b}\bar{b}$  and  $c\bar{c}\bar{c}\bar{c}$  systems in units of MeV. The masses in the fifth column are estimated with  $m_b = 4630$  MeV and  $m_c = 1430$  MeV. The last column lists masses estimated from the  $(\Upsilon\Upsilon)$  or  $(J/\psi J/\psi)$  threshold. The base for the  $J = 0$  case is  $(\phi_2\chi_3, \phi_1\chi_6)^T$ .

System	$J^{PC}$	$\langle H_{CM} \rangle$	Eigenvalue	Eigenvector	Mass	$(\Upsilon\Upsilon)/(\psi\psi)$
$(b\bar{b}\bar{b}\bar{b})$	$2^{++}$	32.0	32.0	1	18552	18920
	$1^{+-}$	0.0	0.0	1	18520	18889
	$0^{++}$	$\begin{pmatrix} -16.0 & 58.8 \\ 58.8 & 24.0 \end{pmatrix}$	$\begin{bmatrix} 66.1 \\ -58.1 \end{bmatrix}$	$\begin{bmatrix} (0.58, 0.81) \\ (-0.81, 0.58) \end{bmatrix}$	$\begin{bmatrix} 18586 \\ 18462 \end{bmatrix}$	$\begin{bmatrix} 18955 \\ 18831 \end{bmatrix}$
$(c\bar{c}\bar{c}\bar{c})$	$2^{++}$	56.5	56.5	1	5777	6194
	$1^{+-}$	0	0	1	5720	6137
	$0^{++}$	$\begin{pmatrix} -28.3 & 103.9 \\ 103.9 & 42.4 \end{pmatrix}$	$\begin{bmatrix} 116.8 \\ -102.6 \end{bmatrix}$	$\begin{bmatrix} (0.58, 0.81) \\ (-0.81, 0.58) \end{bmatrix}$	$\begin{bmatrix} 5837 \\ 5617 \end{bmatrix}$	$\begin{bmatrix} 6254 \\ 6035 \end{bmatrix}$

The decay modes are helpful to the search for these states at experiments. The thresholds of  $\Upsilon\Upsilon$ ,  $\eta_b\Upsilon$ , and  $\eta_b\eta_b$  are 18921 MeV, 18857 MeV, and 18793 MeV, respectively. Those of  $\psi\psi$ ,  $\eta_c\psi$ , and  $\eta_c\eta_c$  are 6194 MeV, 6080 MeV, and 5966 MeV, respectively. The decays into these channels in the first approach of estimation are all kinematically forbidden. In the second approach, the decay through quark rearrangement is possible. Since the feature for the  $c\bar{c}\bar{c}\bar{c}$  system is very similar to the bottom case, we here only concentrate on the latter one.

For the  $J = 2$  tetraquark, the present model calculation gives  $M \approx m_\Upsilon + m_\Upsilon$  and it is difficult to reach a conclusion whether the  $\Upsilon\Upsilon$  decay channel is open. For the  $J = 1$  tetraquark, its mass is 32 MeV above the  $\eta_b\Upsilon$  threshold. So it



can decay into these two mesons. One expects a broader width for such a tetraquark. For the two scalar tetraquarks, one finds that the mixing between different color structures is important, which enlarges the mass difference between the two states. If one does not consider the mixing, the masses are 18913 MeV and 18873 MeV. The higher state is slightly below the  $\Upsilon\Upsilon$  threshold. Once the mixing is considered, the higher state ( $6_c bb$  dominates) can decay into both  $\Upsilon\Upsilon$  and  $\eta_b\eta_b$  channels while the lower one ( $3_c bb$  dominates) decays only into  $\eta_b\eta_b$ .

If the second approach is correct, one expects several resonances around the threshold of two bottomonia. If a resonance is observed in the  $\Upsilon\Upsilon$  channel, this state might be a tetraquark and one may easily identify its spin by measuring its  $\eta_b\eta_b$  decay channel.

### C. The $bb\bar{c}\bar{c}$ and $cb\bar{b}\bar{b}$ systems

We give the numerical results in Table II. The mass splitting between different spins is less than 140 MeV. This number lies between the splittings for the  $bb\bar{b}\bar{b}$  case and the  $cc\bar{c}\bar{c}$  case. The first method of estimation gives a lower mass than the second one. The mass difference is about 540 MeV. To discuss the decay properties, one needs the thresholds of  $B_c^*B_c^*$ ,  $B_cB_c^*$ , and  $B_cB_c$ , which are 12691 MeV, 12621 MeV, and 12551 MeV, respectively. Of course, the strong decays into these channels are all kinematically forbidden in the first method.

TABLE II: Results for the  $bb\bar{c}\bar{c}$  and  $cb\bar{b}\bar{b}$  systems in units of MeV. The masses in the fifth column are estimated with  $m_b = 4630$  MeV and  $m_c = 1430$  MeV. The last column lists masses estimated from the  $(B_cB_c)$  threshold. The base for the  $J = 0$  case is  $(\phi_2\chi_3, \phi_1\chi_6)^T$ .

$J^P$	$\langle H_{CM} \rangle$	Eigenvalue	Eigenvector	Mass	$(B_cB_c)$
$2^+$	39.7	39.7	1	12160	12697
$1^+$	4.5	4.5	1	12125	12661
$0^+$	$\begin{pmatrix} -13.1 & 64.7 \\ 64.7 & 33.2 \end{pmatrix}$	$\begin{bmatrix} 78.7 \\ -58.6 \end{bmatrix}$	$\begin{bmatrix} (0.58, 0.82) \\ (-0.82, 0.58) \end{bmatrix}$	$\begin{bmatrix} 12199 \\ 12061 \end{bmatrix}$	$\begin{bmatrix} 12736 \\ 12598 \end{bmatrix}$

According to the estimation in the second approach, the  $J = 2$  tetraquark may decay into  $B_c^*B_c^*$  and the  $J = 1$  state into  $B_cB_c^*$ . Without considering the mixing between the two scalar tetraquarks, the higher (lower) mass is 12690 (12634) MeV. Both of them can decay into  $B_cB_c$ . The mixing of the color-spin structures induces a much higher mass and a much lower mass. Then the decay channel  $B_c^*B_c^*$  is entirely opened for the higher state ( $6_c bb$  or  $cc$  dominates) while no channel is closed for the lower state ( $3_c bb$  or  $cc$  dominates).

Up to now, experiments confirm only the ground  $B_c$  meson. That means only the scalar tetraquarks decaying into  $B_cB_c$  may be observed in the near future. In the case that the  $B_c^*$  meson is confirmed with enough data, the observations of the other  $bb\bar{c}\bar{c}$  tetraquarks are possible.

### D. The $bbb\bar{c}$ and $cb\bar{b}\bar{b}$ systems

TABLE III: Results for the  $bbb\bar{c}$  and  $cb\bar{b}\bar{b}$  systems in units of MeV. The masses in the fifth column are estimated with  $m_b = 4630$  MeV and  $m_c = 1430$  MeV. The last column lists masses estimated from the  $(\Upsilon B_c)$  threshold. The base for the  $J = 1$  case is  $(\phi_2\chi_2, \phi_2\chi_4, \phi_1\chi_5)^T$  and that for the  $J = 0$  case is  $(\phi_2\chi_3, \phi_1\chi_6)^T$ .

$J^P$	$\langle H_{CM} \rangle$	Eigenvalue	Eigenvector	Mass	$(\Upsilon B_c)$
$2^+$	33.6	33.6	1	15354	15806
$1^+$	$\begin{pmatrix} 0.0 & -1.1 & 2.4 \\ -1.1 & -18.4 & -35.6 \\ 2.4 & -35.6 & 7.6 \end{pmatrix}$	$\begin{bmatrix} 32.7 \\ -0.2 \\ -43.3 \end{bmatrix}$	$\begin{bmatrix} (0.07, -0.57, 0.82) \\ (1.00, 0.05, -0.06) \\ (0.01, -0.82, -0.57) \end{bmatrix}$	$\begin{bmatrix} 15353 \\ 15320 \\ 15277 \end{bmatrix}$	$\begin{bmatrix} 15805 \\ 15773 \\ 15729 \end{bmatrix}$
$0^+$	$\begin{pmatrix} -16.8 & 61.7 \\ 61.7 & 25.2 \end{pmatrix}$	$\begin{bmatrix} 69.4 \\ -61.0 \end{bmatrix}$	$\begin{bmatrix} (0.58, 0.81) \\ (-0.81, 0.58) \end{bmatrix}$	$\begin{bmatrix} 15389 \\ 15259 \end{bmatrix}$	$\begin{bmatrix} 15842 \\ 15712 \end{bmatrix}$

From the results presented in Table III, the maximum mass splitting is around 130 MeV. The mass difference between the estimations with two approaches is about 450 MeV. The thresholds for the possible decay channels  $\Upsilon B_c$ ,  $\Upsilon B_c^*$ ,  $\eta_b B_c$ , and  $\eta_b B_c^*$  are 15736 MeV, 15806 MeV, 15672 MeV, and 15742 MeV, respectively. Again the tetraquarks in the first method do not decay through quark rearrangements while such decays in the second method are possible.

For the  $J = 2$  tetraquark, the situation,  $M \approx m_\Upsilon + m_{B_c^*}$ , is similar to the  $J = 2$   $bb\bar{b}\bar{b}$  or  $cc\bar{c}\bar{c}$  tetraquark case. Maybe it can decay marginally into  $\Upsilon B_c^*$ . For the three  $J = 1$  states, the color-spin mixing affects the intermediate state little while it makes the mass for the high (low) state upward (downward) 25 MeV. The resulting observation is: the high state may decay into  $\Upsilon B_c^*$  (marginally),  $\Upsilon B_c$ , and  $\eta_b B_c^*$ , the intermediate state into  $\Upsilon B_c$  and  $\eta_b B_c^*$ , while the low state does not have a rearrangement decay mode. For the two  $J = 0$  tetraquarks, they may both decay into  $\eta_b B_c$  and the higher state may also decay into  $\Upsilon B_c^*$  after mixing effect is considered.

From the above results, the resonance structures in the  $\Upsilon B_c$ ,  $\eta_b B_c$  channels would probably be observed once experiments collect enough  $B_c$  data.

### E. The $cc\bar{c}\bar{b}$ and $bc\bar{c}\bar{c}$ systems

The color-spin structure is the same as the  $bb\bar{b}\bar{c}$  system but the decay feature relies on masses and may be different. We present the numerical results in Table IV. The maximum mass splitting around 180 MeV is between the two scalar tetraquarks. Similar to the  $bb\bar{b}\bar{c}$  system, the second method gives a 450 MeV higher estimation for the masses. Here the relevant meson-meson thresholds are  $\psi B_c$  (9373 MeV),  $\psi B_c^*$  (9443 MeV),  $\eta_c B_c$  (9259 MeV), and  $\eta_c B_c^*$  (9329 MeV). Rearrangement decays for these tetraquarks are possible only when the mass is high enough.

The decay for the  $J = 2$  tetraquark into  $\psi B_c^*$  is marginal. For the three axial vector tetraquarks, the low mass one has only one decay channel  $\eta_c B_c^*$ , the intermediate one has two  $\eta_c B_c^*$  and  $\psi B_c$ , and the high mass state has one more channel  $\psi B_c^*$ . The decays for the two  $J = 0$  states into  $\eta_c B_c$  are both allowed while the channel  $\psi B_c^*$  is also opened for the higher tetraquark.

Early investigations on possible resonances in the  $cc\bar{c}\bar{b}$  system should be through the channels  $\psi B_c$  and  $\eta_c B_c$ , which means that a high mass axial vector and two scalar tetraquarks could be observed first.

TABLE IV: Results for the  $cc\bar{c}\bar{b}$  and  $bc\bar{c}\bar{c}$  systems in units of MeV. The masses in the fifth column are estimated with  $m_b = 4630$  MeV and  $m_c = 1430$  MeV. The last column lists masses estimated from the  $(\psi B_c)$  threshold. The base for the  $J = 1$  case is  $(\phi_2\chi_2, \phi_2\chi_4, \phi_1\chi_5)^T$  and that for the  $J = 0$  case is  $(\phi_2\chi_3, \phi_1\chi_6)^T$ .

$J^P$	$\langle H_{CM} \rangle$	Eigenvalue	Eigenvector	Mass	$(\psi B_c)$
$2^+$	45.9	45.9	1	8966	9443
$1^+$	$\begin{pmatrix} 0.0 & -7.5 & 16.0 \\ -7.5 & -12.3 & -48.6 \\ 16.0 & -48.6 & 16.8 \end{pmatrix}$	$\begin{bmatrix} 58.2 \\ -4.9 \\ -48.8 \end{bmatrix}$	$\begin{bmatrix} (0.29, -0.56, 0.77) \\ (0.96, 0.23, -0.18) \\ (0.08, -0.79, -0.61) \end{bmatrix}$	$\begin{bmatrix} 8978 \\ 8915 \\ 8871 \end{bmatrix}$	$\begin{bmatrix} 9455 \\ 9392 \\ 9348 \end{bmatrix}$
$0^+$	$\begin{pmatrix} -22.9 & 84.3 \\ 84.3 & 34.4 \end{pmatrix}$	$\begin{bmatrix} 94.7 \\ -83.3 \end{bmatrix}$	$\begin{bmatrix} (0.58, 0.81) \\ (-0.81, 0.58) \end{bmatrix}$	$\begin{bmatrix} 9015 \\ 8837 \end{bmatrix}$	$\begin{bmatrix} 9492 \\ 9314 \end{bmatrix}$

### F. The $bc\bar{b}\bar{c}$ system

We show the results for the twelve states in Table V. Now the maximum mass splitting (240 MeV) again occurs between the scalar tetraquarks while the mass difference between two estimation approaches can reach 540 MeV, depending on the reference threshold. Since we calculate only mass splitting with one part of the gluon exchange interaction, the absolute values of mass cannot be obtained. Future full calculation may answer which scheme is more reasonable. Seven meson-meson channels are involved in discussing decay properties with the obtained results. The channels (thresholds) are  $\Upsilon\psi$  (12557 MeV),  $\Upsilon\eta_c$  (12443 MeV),  $\eta_b\psi$  (12493 MeV),  $\eta_b\eta_c$  (12379 MeV),  $B_c^{*-}B_c^{*+}$  (12691 MeV),  $B_c^-B_c^{*+}$  (12621 MeV), and  $B_c^-B_c^+$  (12551 MeV). We now take a look at the rearrangement decays with the masses estimated in the second approach.

First, we use the threshold of  $\Upsilon\psi$  as a reference. One of the  $J = 2$  tetraquarks is around this threshold and the other is slightly below it. The rearrangement decay for the higher one is just marginal. Their decays into  $B_c^{*-}B_c^{*+}$  are kinematically forbidden. So the useful channel to search for a tensor tetraquark is only  $\Upsilon\psi$ . The two  $1^{++}$  states have similar decay properties as the tensor tetraquarks, which indicates that it is difficult to identify the spin of a state observed in  $\Upsilon\psi$  without enough data. The open-bottom-charm decay channels for the four  $1^{+-}$  tetraquarks are all kinematically forbidden. Three of them can decay into  $\Upsilon\eta_c$  and the highest two can also decay into  $\eta_b\psi$ . For the four  $0^{++}$  states, three of them can decay into  $\eta_b\eta_c$  while the other's rearrangement decay is forbidden. The decays into  $\Upsilon\psi$  and  $B_c^-B_c^+$  for the highest tetraquark are also allowed.



Secondly, we use the threshold of  $B_c^- B_c^+$  as a reference. Now all the masses are higher. The decays for the higher  $2^{++}$  state into  $\Upsilon\psi$  and  $B_c^{*-} B_c^{*+}$  are allowed. The decay for the lower one into  $\Upsilon\psi$  is allowed while into  $B_c^{*-} B_c^{*+}$  is marginal. The higher  $1^{++}$  tetraquark decays into both  $\Upsilon\psi$  and  $B_c^- B_c^{*+} + B_c^{*-} B_c^+$  while the lower one into  $\Upsilon\psi$  only. For the four  $1^{+-}$  states, their decays into  $\Upsilon\eta_c$  and  $\eta_b\psi$  are all allowed. Open-bottom-charm channel  $B_c^- B_c^{*+} - B_c^{*-} B_c^+$  is also allowed for the highest two and marginally for the state with the mass 12620 MeV. The highest one can also marginally decay into  $B_c^{*-} B_c^{*+}$ . The allowed rearrangement decay channels for the four  $0^{++}$  tetraquarks are:  $\Upsilon\psi$ ,  $\eta_b\eta_c$ ,  $B_c^{*-} B_c^{*+}$ , and  $B_c^- B_c^+$  for the highest one,  $\Upsilon\psi$ ,  $\eta_b\eta_c$ , and  $B_c^- B_c^+$  for states with mass 12655 MeV and 12584 MeV, and only  $\eta_b\eta_c$  for the lowest one.

TABLE V: Results for the  $b\bar{c}b\bar{c}$  system in units of MeV. The masses in the fifth column are estimated with  $m_b = 4630$  MeV and  $m_c = 1430$  MeV. The last two columns list masses estimated from the  $(\Upsilon\psi)$  and the  $(B_c B_c)$  thresholds. The bases for the  $J = 2$  and  $J = 0$  cases are  $(\phi_1\chi_1, \phi_2\chi_1)^T$  and  $(\phi_1\chi_3, \phi_1\chi_6, \phi_2\chi_3, \phi_2\chi_6)^T$ , respectively. If  $J = 1$ , the bases for the cases  $C = +$  and  $C = -$  are  $([\phi\chi]_+^{66}, [\phi\chi]_+^{33})^T$  and  $(\phi_1\chi_2, \phi_2\chi_2, [\phi\chi]_-^{66}, [\phi\chi]_-^{33})^T$ , respectively.

$J^{PC}$	$\langle H_{CM} \rangle$	Eigenvalue	Eigenvector	Mass	$(\Upsilon\psi)$	$(B_c B_c)$
$2^{++}$	$\begin{pmatrix} 40.9 & 4.8 \\ 4.8 & 37.5 \end{pmatrix}$	$\begin{bmatrix} 44.3 \\ 34.1 \end{bmatrix}$	$\begin{bmatrix} (0.82, 0.58) \\ (0.58, -0.82) \end{bmatrix}$	$\begin{bmatrix} 12164 \\ 12154 \end{bmatrix}$	$\begin{bmatrix} 12557 \\ 12547 \end{bmatrix}$	$\begin{bmatrix} 12701 \\ 12691 \end{bmatrix}$
		$\begin{bmatrix} 79.2 \\ -2.2 \\ -73.2 \\ -160.5 \end{bmatrix}$	$\begin{bmatrix} (-0.02, 0.81, 0.58, -0.03) \\ (0.57, -0.01, 0.08, 0.81) \\ (0.08, -0.58, 0.80, -0.14) \\ (0.82, 0.09, -0.12, -0.56) \end{bmatrix}$	$\begin{bmatrix} 12199 \\ 12118 \\ 12045 \\ 11960 \end{bmatrix}$	$\begin{bmatrix} 12592 \\ 12511 \\ 12440 \\ 12352 \end{bmatrix}$	$\begin{bmatrix} 12736 \\ 12655 \\ 12584 \\ 12496 \end{bmatrix}$
$1^{++}$	$\begin{pmatrix} 14.5, -42.1 \\ -42.1, -15.3 \end{pmatrix}$	$\begin{bmatrix} 44.3 \\ -45.1 \end{bmatrix}$	$\begin{bmatrix} (-0.82, 0.58) \\ (0.58, 0.82) \end{bmatrix}$	$\begin{bmatrix} 12164 \\ 12075 \end{bmatrix}$	$\begin{bmatrix} 12557 \\ 12468 \end{bmatrix}$	$\begin{bmatrix} 12701 \\ 12612 \end{bmatrix}$
		$\begin{bmatrix} 36.7 \\ -0.3 \\ -36.6 \\ -77.3 \end{bmatrix}$	$\begin{bmatrix} (0.04, -0.17, -0.80, -0.57) \\ (0.01, 0.96, 0.00, -0.29) \\ (0.68, 0.17, -0.42, 0.59) \\ (-0.74, 0.16, -0.43, 0.50) \end{bmatrix}$	$\begin{bmatrix} 12157 \\ 12120 \\ 12083 \\ 12043 \end{bmatrix}$	$\begin{bmatrix} 12550 \\ 12513 \\ 12476 \\ 12436 \end{bmatrix}$	$\begin{bmatrix} 12694 \\ 12657 \\ 12620 \\ 12580 \end{bmatrix}$
$1^{+-}$	$\begin{pmatrix} -58.5 & 4.8 & -15.3 & 13.0 \\ 4.8 & -2.3 & 13.0 & -6.1 \\ -15.3 & 13.0 & 3.1 & 42.1 \\ 13.0 & -6.1 & 42.1 & -19.9 \end{pmatrix}$	$\begin{bmatrix} 36.7 \\ -0.3 \\ -36.6 \\ -77.3 \end{bmatrix}$	$\begin{bmatrix} (0.04, -0.17, -0.80, -0.57) \\ (0.01, 0.96, 0.00, -0.29) \\ (0.68, 0.17, -0.42, 0.59) \\ (-0.74, 0.16, -0.43, 0.50) \end{bmatrix}$	$\begin{bmatrix} 12157 \\ 12120 \\ 12083 \\ 12043 \end{bmatrix}$	$\begin{bmatrix} 12550 \\ 12513 \\ 12476 \\ 12436 \end{bmatrix}$	$\begin{bmatrix} 12694 \\ 12657 \\ 12620 \\ 12580 \end{bmatrix}$

Based on the above results, one may use  $\eta_b\eta_c$  and  $B_c^- B_c^+$  channels to search for possible scalar tetraquarks, and use  $\Upsilon\eta_c$  and  $\eta_b\psi$  channels for  $1^{+-}$  states. The  $\Upsilon\psi$  channel may be searched for, but the identification for the tetraquark's spin, 0, 1, or 2, needs enough experimental data.

#### IV. SUMMARY

In the chiral quark model, the interaction between the light quarks may also arise from the exchange of Goldstone bosons. For the pure heavy systems, such an interaction is absent. Therefore, one needs to consider only the gluon-exchange interaction for the present  $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$  system. As a short range force, it tends to form the compact tetraquarks rather than the meson-meson molecules. We have studied the mass splitting of these tetraquark states with the color-magnetic interaction in this work.

For the case ( $b\bar{c}b\bar{c}$  system) without constraint from the Pauli principle, the number of color-singlet tetraquarks with  $6_c$  diquark is equal to that with  $\bar{3}_c$  diquark. The two color structures couple through the color-magnetic interaction. From Table V, the coupling for the  $2^{++}$  states is weak while that for the  $1^{++}$  states is stronger. For the cases with constraint from the Pauli principle, the number of the color-singlet tetraquarks with the  $\bar{3}_c$  diquark is bigger than that with  $6_c$  diquark. Their mixing is generally significant (see Tables I-IV). In both cases, the tetraquarks with  $6_c$  diquark do not exist independently.

After the configuration mixing effects are considered, both the heaviest tetraquark and the lightest tetraquark for a system are the scalar states. The mass differences are 124 MeV, 220 MeV, 138 MeV, 130 MeV, 178 MeV, and 240 MeV for the  $bb\bar{b}\bar{c}$ ,  $cc\bar{c}\bar{b}$ ,  $bb\bar{c}\bar{c}$ ,  $bb\bar{b}\bar{c}$ ,  $cc\bar{c}\bar{b}$ , and  $b\bar{c}b\bar{c}$  systems, respectively. Other states fall within these mass difference ranges. Whether the tetraquarks decay through quark rearrangements rely on their masses.

For comparison, we estimate the masses with two approaches. In the first approach, the phenomenological effective quark masses  $m_c = 1430$  MeV and  $m_b = 4630$  MeV are used. The resulting tetraquarks have relatively low masses and do not have rearrangement decay channels. In the second approach, we use the threshold of some meson-meson state as a reference. One finds several tetraquarks that rearrangement decays are kinematically forbidden. Higher tetraquarks may be observed in channels like  $\Upsilon\Upsilon$ ,  $\eta_b\eta_b$ , and so on. Some states have marginal decay channels and exotic structures around such thresholds might be possible. For the  $b\bar{c}b\bar{c}$  system, there are two different thresholds

and estimated masses are different accordingly. The present simple model needs to be improved to give more reliable results.

If experiments could observe one resonant state in the channel of two heavy quarkonia, its nature as a tetraquark is favored. More tetraquarks should also exist and searches for them are strongly called for. We hope the decay channels discussed in this paper are helpful for the experimental search.

To summarize, we have explored the mass splitting between the  $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$  tetraquarks with the color-magnetic interaction. The mixing between different color structures are considered. We have estimated their masses with two approaches and discusses possible rearrangement decay channels. Hopefully the exotic tetraquark states composed of four heavy quarks may be observed at LHCb and BelleII in the future.

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